Department of Mathematics
Punjabi University, Patiala

# Ph.D. Entrance Syllabus
2020

## Section A
**(Research Methodology)**

1. **Research Aptitude and Research Components**: Meaning of Research, Objective of Research, Motivation in Research, Types of Research, Research Approaches, Significance of Research, Research Methods versus Methodology, Research and Scientific Method, Research Process, Criteria of Good Research, Defining the Research problem. What is a research problem? Selecting the research problem, Necessity of Defining the problem, Techniques involved in defining a problem, Research design, Meaning of Research design. Need for good research design, Research methodology, Features of a good design, Important concepts relating to research design, Different research designs, Introduction to Thesis writing, Research project writing: Characteristics and Format.

2. **Mathematical and Logical Reasoning:** Number Series, letter series, codes, relationships, classification. Understanding the structure of argument, Evaluating and distinguishing deductive and inductive reasoning. Verbal analogies, word analogies-applied analogy, verbal classification, Reasoning logical diagrams, simple diagrammatic relationship, multi-diagrammatic relationship. Venn-diagrams, analytic reasoning.

3. **Data Interpretation and Basic Statistical Techniques:** Sources, acquisition and interpretation of data, quantitative and qualitative data, graphical representation and mapping of data, basic concepts of probability, discrete distributions and continuous distributions with applications.

4. **Fundamental of computer science**: ICT: meaning, advantages, disadvantages and users, general abbreviation and terminology, basics of internet and emailing, MS-office (MS WORD, MS Excel and Power Point).

## Section B
## (Subject Specific)

**Algebra:-** Group Action, Class Equations, Sylow's theorem, Fundamental theorem of finitely generated Abelian groups, Solvable and Nilpotent Groups, Automorphism group of cyclic groups, Ideals of Matrix Rings, Factorization in Integral Domains, Modules, Free Modules, Vector spaces, subspaces, Basis, Eigen values, Eigen vectors, Jordan Cannonical form, Rational Cannonical form, Noetherian, Artinian Modules, Field Extensions, Normal and seperable extensions, Algebraically closed fields, Perfect fields, Finite fields, Fundamental theorem of Galois theory, Cyclotomic Extensions, Solvability by Radicals.

**Topology: -** Cardinals topological spaces, Elementary concepts: Interior, Closure and Derived Set. Relativisation. Continuity and its Characterizations, Cartesian product topology, Connectedness, Compactness, Invariance of Connectedness and Compactness under arbitrary products. Relationship of Compactness with Nets and Filters. Countability: First and Second countable spaces. Lindelof spaces, Equality of Compactness, Countable Compactness and the Heine Borel Theorem in Metric spaces. Identification Topology and Quotient Spaces, One Point Compactification and the Stone Czech Compactification .

**Analysis:-** Metric space, Riemann-Stieljes integration, Power series, Fourier series, Linear transformations, Taylor's theorem, Inverse function theorem, Implicit function theorem. General measurable spaces, Lebesgue measure, Measurable functions w.r.t. general measure, Integration theorem, Integration of series. Riemann and Lebesgue integrals. Banach spaces, Hahn-Banach theorem in Linear Spaces and its applications, Uniform boundedness principle, Open mapping theorem, Closed graph theorem, C[a,b], Reflexivity. Hilbert spaces, Adjoint operators, Self-adjoint operators, Normal and unitary operators. Analytic function, Cauchy-Riemann equations, Conformal Mapping, Complex Integration, Taylor's theorem. Laurent series in an annulus. Singularities, Argument principle, Rouche's theorem, Fundamental theorem of Algebra, Maximum Modulus Principle, Residue calculus.

**Differential Geometry:-** Curves in the plane and space, Surfaces in three dimensions, the lengths of curves on a surface, the first fundamental theorem, curvature of surfaces, the second fundamental form, the Gauss and Weingarten maps, Normal and Geodesic curvatures, Gaussian and Principal curvatures, Geodesics, Gauss remarkable theorem , Surfaces of constant Gaussian curvature.

**Differential Equations:-** Existence and Uniqueness Theorem and applications of ODE of first order and higher order system of differential equations. Fundamental set of solutions, Fundamental Matrix, Wronskian, Abel Liouville formula, Reduction of order, Adjoint systems and self adjoint systems of second order.

**Numerical solution of ODE and PDE:** Numerical solution of ordinary and partial differential equations; Euler's method, Improved Euler's method, RK method of $4^{th}$ order, Finite difference

methods, Stiff equations, Milne's method, Adams-Bashforth method, Numerical solution of Laplace equations, Wave equations and Heat equations.

**Mathematical Methods:-** Integral solutions of Fredhlom's equation, Variational problems and its solutions.

**Solid Mechanics:-** Tensor analysis, analysis of strain, analysis of stress, Hooke's Law, Airy's stress function.

**Optimization:-** Formulation of the general LPP, simplex method, duality theory, assignment and transportation problem, integer programming, game theory, queueing theory, inventory theory, replacement problem, Network analysis.